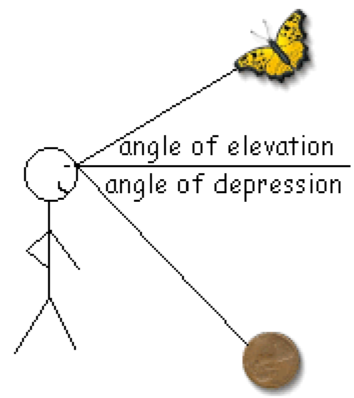
GOLDen Mathematics: Trigonometry

Right Triangle Trig Applications

Hi everyone! Today we are ready to explore even more real life trig applications! This is where the fun begins ;-}

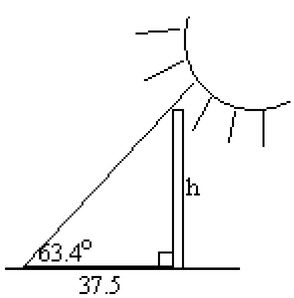
Angles of Elevation and Depression:

A line of sight is a line drawn horizontally out from the eyes as if looking straight forward. It is off this line that we will reference. The person at the right looks up at a butterfly. The positive acute angle between the line of sight and the line to the butterfly is called the angle of elevation. Next the person looks down and sees a penny (find a penny, pick it up, all the day you'll have good luck!). The positive acute angle between the line of sight and the line to the coin is called the angle of depression.



Let's try a problem together.

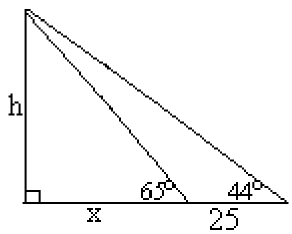
Example 1. Suppose that when the sun is at an angle of elevation in the sky of 63.4°, a building casts a shadow of 37.5 feet. How tall is the building?



Since we have the length of the side next to the given angle and we want the side across from the angle, we'll choose a trig function that uses the adj and opp,

$\tan 63.4° = h/37.5$
multiplying through by 37.5 gives,
$37.5\tan 63.4° = h$
rounding to 3 sig digits gives $h = 74.9$ ft

Example 2. **a systems of trig equations example.** Two friends are having a trig play day and decide to try to estimate the height of a tree. They position themselves 25 feet apart on the same side of and in a straight line with a big oak. Pressing their faces to the ground so their line of sight is the ground, they look up at the top of the tree and estimate their angles of elevation to the top of the tree as 44° and 65°. Now take this information and help them find the height of the tree.



Note there are two right triangles. From each we can set up a tangent function, $\tan 65° = h/x$ and $\tan 44° = h/(x+25)$. Since we have two equations in two unknowns, we must solve a system. (Remember those?) Let's solve by the substitution method.

Solving the first equation for h we get,

$$h = x \tan 65°$$

plugging that into the second equation gives,

$$\tan 44° = x \tan 65° / (x+25)$$

from here we do a bunch of algebra,

$$(x + 25) \tan 44° = x \tan 65°$$

$$x \tan 44° + 25 \tan 44° = x \tan 65°$$

$$25 \tan 44° = x \tan 65° - x \tan 44°$$

$$25 \tan 44° = x(\tan 65° - \tan 44°)$$

$$25 \tan 44° / (\tan 65° - \tan 44°) = x$$

But, we want h! So, plugging this back in for h above,

$$h = [25 \tan 44° / (\tan 65° - \tan 44°)] \tan 65°$$

plugging all this into a calculator and rounding to two sig digits gives,

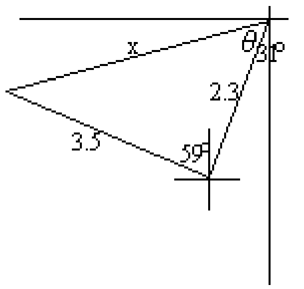
finally, $h = 44$ feet.

Bearings:

Direction can be given in many ways. One is by stating your bearing. The bearing of a line is the acute angle formed by the line and the north-south line. The notation begins with N or S (depending from which you are referencing) followed by the number of degrees in the angle and ends with E or W (depending on the direction the angle goes from the N-S line). E.g., N65°W read "north 65 degrees west" or "65 degrees west of north" means a line in quadrant II 65° off the positive y-axis.

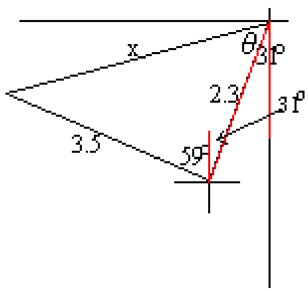
Let's try a problem.

Example 3. A man boating in a large lake goes 2.3 miles from the dock in the direction S31°W. He turns the boat to a bearing of N59°W and travels 3.5 miles. How far is he then and what is his bearing from the dock?



Notice in the figure that after the boat goes 2.3 miles in direction S31°W, then he turns and we need a new reference, so I drew in new axes. Then he goes N59°W for 3.5 miles. x represents his distance back to the dock. If we can find θ, we can write his final position bearing with respect to the south line.

One problem. Do we have a right triangle? Without it we are up a creek!



This is where a nifty theorem from geometry comes in. Notice that the two y-axes are parallel and the first 2.3 mile line connects them. The "alternate interior angle" theorem says that the angle pointed to by the arrow must match the 31° angle. I call this the <u>Zorro rule</u> (see the big red Z?). Since 31°+59°=90°, we do have a 90° turn!

So, $\tan\theta = 3.5/2.3 \Rightarrow \theta = \tan^{-1}(3.5/2.3) \approx 57°$
Thus his bearing to the dock is S88°W.
$x = \text{sqrt}(3.5^2 + 2.3^2) \approx 4.2$ miles.

Are you enjoying yourself? These applications are getting quite involved and sophisticated. Drill and practice problems are necessary to get the basics down, but applying our foundation of knowledge to real-world applications is really the purpose of the course and where the real learning occurs. Have fun!