

Applications of Factoring Polynomials

Hello everyone. As you know, we are able to solve polynomial equations by factoring. Polynomial functions have widespread applications in business, life science, medicine, and engineering, so it is imperative to learn the factoring methods well. Let's look at some specific applications.

I admit I don't go through my day having to factor polynomials (except for fun while waiting in a line at the bank!) But polynomial models are used in economics, engineering, biology, and sociology, just to name a few. Usually these polynomials are functions that need to be evaluated for specific parameters or they are equations that must be solved for x . This is where factoring is particularly useful. The factored form of a polynomial equation can tell you things that the expanded form cannot.

Application from engineering. When standing on the Golden Gate Bridge in San Francisco, California, you are 220 feet above the mean water level. If you drop your camera, its position above the water can be given by the polynomial equation $H=220-16t^2$ where H is how many feet above the water the camera is at time t seconds after you drop it. When will it hit the water? It's hard to tell in this form, but if we factor the equation into $H = 220 - 16t^2 = -16(t^2 - 13.75) \approx -16(t-3.7)(t+3.7)$ then we can immediately tell that 3.7 seconds after dropping the camera it will hit the water (i.e. $H=0$). [This example is adapted from Blitzer, Robert. Introductory Algebra. 3rd ed. New York: Prentice Hall, 2001.]

Application from business. My husband was a manager in a production firm that makes cat furniture. Part of his job was to determine how a change in price in one of the models might affect sales. Costco was one of their largest buyers. The demand equation for the Cat Condo model 67 is $p=40.14-0.000566x$ where p is the price in U.S. dollars and x is the number of units produced and sold at that price. If the price goes up, of course the number of units Costco would buy would go down. If the price is \$32.50, how many units will be sold? To figure this out we must set $p=32.50$ and solve for x . Since this is just a linear equation, you should have no trouble finding that $x \approx 13,500$.

The cost equation for the Cat Condo model 67 is $C=15.25x+125000$ where C is the total cost in U.S. dollars to produce x units. The total profit obtained by producing and selling x units of Cat Condo model 67 is Profit = Revenue - Cost = (price)(# units sold) - Cost = $px - C = x(40.14-0.000566x)-(15.25x+125000) = -0.000566x^2 + 24.89x - 125000$. It would be easy to evaluate this trinomial

for x , e.g. what is the profit from the production and sale of 10000 units?
(Answer: \$67,300) But what if the company owner wanted to see a profit of \$100,000? How many units would need to be sold? What price would need to be set so that Costco would want to buy this many units? We aren't able to fully answer these questions (yet!) but we can set the problem up and answer them after another few lessons ;-). The main point is that we need to be able to factor polynomial expressions before we can solve polynomial equations.

$$P = -0.000566x^2 + 24.89x - 125000$$

$$100000 = -0.000566x^2 + 24.89x - 125000$$

$$0 = -0.000566x^2 + 24.89x - 125000 - 100000$$

$$0 = -0.000566x^2 + 24.89x - 225000$$

$$0 = (\quad) (\quad) \text{ yucky, but possible!}$$

Application from biology. For this predator-prey application I direct you to an "Ask Dr Math" posting at mathforum.org/library/drmath/view/53698.html. The problem involves a little calculus, but without understanding all the details you can see the usefulness of factoring once again to solve polynomial equations.

I hope these real-world applications will provide you with the motivation to learn to factor well and the determination to stay tuned for when we cover solving polynomial equations by factoring coming soon to a website near you!